Computations of Global Seismic Wave Propagation in Three Dimensional Earth Model

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Abstract. We use a Spectral-Element Method implemented on the Earth Simulator in Japan to simulate broadband seismic waves generated by various earthquakes. The spectral-element method is based on a weak formulation of the equations of motion and has both the flexibility of a finite-element method and the accuracy of a pseudospectral method. The method has been developed on a large PC cluster and optimized on the Earth Simulator. We perform numerical simulation of seismic wave propagation for a three-dimensional Earth model, which incorporates 3D variations in compressional wave velocity, shear-wave velocity and density, attenuation, anisotropy, ellipticity, topography and bathymetry, and crustal thickness. The simulations are performed on 4056 processors, which require 507 out of 640 nodes of the Earth Simulator. We use a mesh with 206 million spectral-elements, for a total of 13.8 billion global integration grid points (i.e., almost 37 billion degrees of freedom). We show examples of simulations for several large earthquakes and discuss future applications in seismological studies.

Keywords: seismic wave propagation, 3–D Earth models, spectral-element method.

1 Introduction

Accurate modeling of seismic wave propagation in fully three-dimensional (3–D) Earth models is of considerable interest in seismology in order to determine both the 3–D seismic-wave velocity structure of the Earth and the rupture process during large earthquakes. However, significant deviations of Earthfs internal structure from spherical symmetry, such as the 3–D seismic-wave velocity structure inside the solid mantle and laterally heterogeneous crust at the surface of the Earth, have made applications of analytical approaches to this problem a formidable task. The numerical modeling of seismic-wave propagation in 3–D structures has been significantly advanced in the last few years due to the introduction of the Spectral-Element Method (SEM), which is a high-degree version of the finite-element method that is very accurate for linear hyperbolic problems such as wave propagation. The 3–D SEM was first used in seismology for local

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and regional simulations [1]–[3], and more recently adapted to wave propagation at the scale of the full Earth [4]–[7].

Here we show that our implementation of the SEM on the Earth Simulator in Japan allows us to calculate theoretical seismic waves which are accurate up to 3.5 seconds and longer for fully 3–D Earth models. We include the full complexity of the 3–D Earth in our simulations, i.e., a 3–D seismic wave velocity [8] and density structure, a 3–D crustal model [9], ellipticity as well as topography and bathymetry. Synthetic waveforms at such high resolution (periods of 3.5 seconds and longer) allow us to perform direct comparisons of arrival times of various body-wave phases between observed and synthetic seismograms, which has never been accomplished before. Usual seismological algorithms, such as normal-mode summation techniques that calculate quasi-analytical synthetic seismograms for one-dimensional (1-D) spherically symmetric Earth models [10], are typically accurate down to 8 seconds [11]. In other words, the SEM on the Earth Simulator allows us to simulate global seismic wave propagation in fully 3–D Earth models at periods shorter than current seismological practice for simpler 1-D spherically symmetric models.

The results of our simulation show that the synthetic seismograms calculated for fully 3–D Earth models by using the Earth Simulator and the SEM agree well with the observed seismograms, which illustrates that the current 3–D seismic velocity model captures the general long-wavelength image of Earth's interior with sufficient resolution.

2 Spectral-Element Method

We use the spectral-element method (SEM) developed by Komatitsch and Tromp (2002a, 2002b) [5,6] to simulate global seismic wave propagation throughout a 3–D Earth model, which includes a 3–D seismic velocity and density structure, a 3–D crustal model, ellipticity as well as topography and bathymetry. The SEM first divides the Earth into six chunks. Each of the six chunks is divided into slices. Each slice is allocated to one CPU of the Earth Simulator. Communication between each CPU is done by MPI. Before the system can be marched forward in time, the contributions from all the elements that share a common global grid point need to be summed. Since the global mass matrix is diagonal, time discretization of the second-order ordinary differential equation is achieved based upon a classical explicit second-order finite-difference scheme.

The maximum number of nodes we could use for this simulation is 4056 processors, i.e., 507 nodes out of 640 of the Earth Simulator. Each slice is allocated to one processor of the Earth Simulator and subdivided with a mesh of 48 48 spec-tral-elements at the surface of each slice. Within each surface element we use $5 \times 5 = 25$ Gauss-Lobatto-Legendre (GLL) grid points to interpolate the wave field [12,13], which translates into an average grid spacing of 2.0 km (i.e., 0.018 degrees) at the surface. The total number of spectral elements in this mesh is 206 million, which corresponds to a total of 13.8 billion global grid points, since each spectral element contains $5 \times 5 \times 5 = 125$ grid points, but with points on its

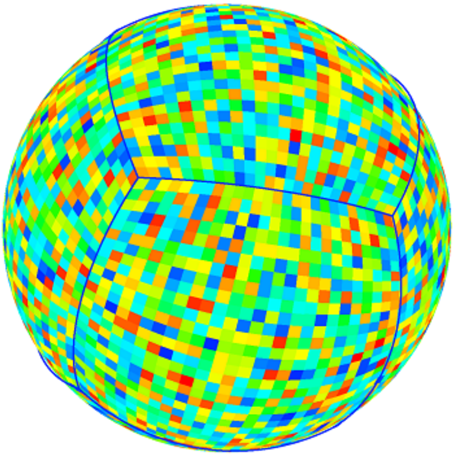

Fig. 1. The SEM uses a mesh of hexahedral finite elements on which the wave field is interpolated by high-degree Lagrange polynomials on Gauss-Lobatto-Legendre (GLL) integration points. This figure shows a global view of the mesh at the surface, illustrating that each of the six sides of the so-called 'cubed sphere' mesh is divided into 26 × 26 slices, shown here with different colors, for a total of 4056 slices (i.e., one slice per processor)

faces shared by neighboring elements. This in turn corresponds to 36.6 billion degrees of freedom (the total num-ber of degrees of freedom is slightly less than 3 times the number of grid points be-cause we solve for the three components of displacement everywhere in the mesh, except in the liquid outer core of the Earth where we solve for a scalar potential). Using this mesh, we can calculate synthetic seismograms that are accurate down to seismic periods of 3.5 seconds. This simulation uses a total of approximately 7 terabytes of memory. Total performance of the code, measured using the MPI Program Runtime Performance Information was 10 teraflops, which is about one third of the expected peak performance for this number of nodes (507 nodes 64gigaflops = 32 teraflops). Figure 1 shows a global view of the spectral-element mesh at the surface of the Earth. In Figure 2, we compare the vertical component of displacement from synthetic seis-mograms calculated using 507 nodes of the Earth Simulator and observed records for several broadband seismic stations of the F-net array operated by the National Institute of Earth Science and Disaster Prevention in Japan. The earthquake we simulated is a deep earthquake of magnitude 6.3 that occurred in South of Japan on November 12, 2003, at a depth of 382 km.

It is surprising that the global 3–D seismic velocity model used in this simulation still produces fairly good agreement with the observations even at periods of 3.5 sec-onds, because it is supposed that the crustal and mantle structure beneath Japanese Islands are highly heterogeneous and are not captured by the long-wavelength global 3D Earth model. However, Figure 2 also shows that the theoretical seismograms calcu-lated with 507 nodes of the Earth Simulator do not reproduce some of the fine features in the observation and suggests the limitation of this global 3–D seismic velocity model.

For those stations located to the north-east of the epicenter (the azimuth is about 20 degrees), the observed waves show large high-frequency oscillations because the waves travel along the subducting pacific plate, but this feature is not modeled in the theoretical seismograms. This shows that we need to improve our 3–D seismic wave velocity model to calculate theoretical seismic waves that are accurate at 3.5 seconds and longer.

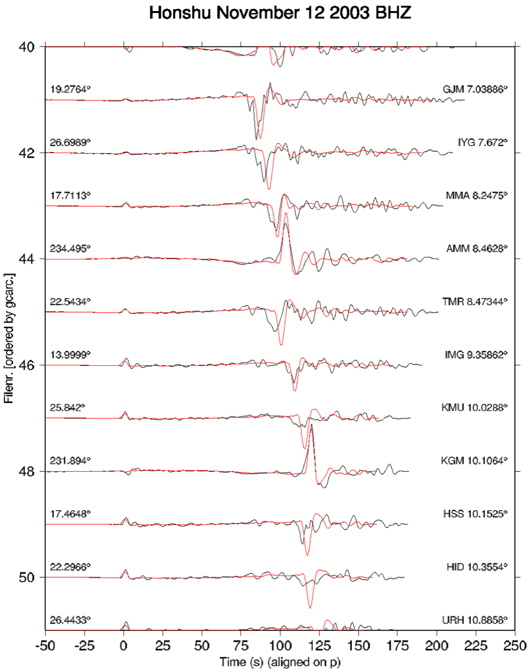

Fig. 2. Broadband data and synthetic displacement seismograms for the 2003 South of Honshu earthquake bandpass-filtered with a two-pass four-pole Butterworth filter between periods of 3.5 and 150 seconds. Vertical component data (black) and synthetic (red) displacement seismograms aligned on the arrival time of the P wave are shown. For each set of seismograms the azimuth is printed above the records to the left, and the station name and epicentral distance are printed to the right.

3 Simulation of the 2004 Sumatra Earthquake

Because we have found that we do not have a 3–D Earth model which has sufficient resolution to simulate seismic wave propagation accurately in regional scale, we de-cide to use 243 nodes (1944 CPUs) of the Earth Simulator for the simulation using the SEM. Using 243 nodes (1944 CPUs), we can subdivide the six chunks into 1944 slices (1944 = 6 × 18 × 18). Each slice is then subdivided into 48 elements in one direction. Because each element has 5 Gauss-Lobatto Legendre integration points, the average grid spacing at the surface of the Earth is about 2.9 km. The number of grid points in total amounts to about 5.5 billion. Using this mesh, it is expected that we can calculate synthetic seismograms accurate up to 5 sec all over the globe. For the 243 nodes case, the total performance we achieved was about 5 teraflops, which also is about one third of the peak performance. The fact that when we double the number of nodes from 243 to 507 the total performance also doubles from 5 teraflops to 10 tera-flops shows that our SEM code exhibits an excellent scaling relation with respect to performance. We calculate synthetic seismograms for a 3–D Earth model using the SEM code and 243 nodes of the ES for the December 26, 2004 Sumatra earthquake (Mw 9.0, depth 15.0 km) in the same manner as Tsuboi et al (2003) [14] and Komatitsch et al (2003) [15], which was awarded 2003 Gordon Bell prize for peak performance in SC2003.

The December 26, 2004 Sumatra earthquake is one of the largest earthquakes ever recorded by modern seismographic instrument. The earthquake started its rupture at the west of northern part of Sumatra Island and propagated in a northwestern direction up to Andaman Islands. Total length of the earthquake fault is estimated to be more than 1000 km and the rupture duration lasts for more than 500 sec. This event has caused devastating tsunami hazard around the Indian Ocean. It is important to know the detailed earthquake fault slip distribution for this earthquake because the excitation mechanism of tsunami is closely related to the earthquake source mechanisms. To simulate synthetic seismograms for this earthquake, we represent the earthquake source by more than 800 point sources distributed both in space and time, which are obtained by seismic wave analysis. In Figure 3, we show snapshots of seismic wave propagation along the surface of the Earth. Because the rupture along the fault propagated in a northwest direction, the seismic waves radiated in this direction are strongly amplified.

This is referred as the directivity caused by the earthquake source mechanisms. Figure 3 illustrate that the amplitude of the seismic waves becomes large in the northwest direction and shows that this directivity is modeled well. Because there are more than 200 seismographic observatories, which are equipped with broadband seismometers all over the globe, we can directly compare the synthetic seismograms calculated with the Earth Simulator and the SEM with the observed seismograms.

Figure 4 shows the results of this comparison for vertical ground motion and demonstrates that the agreement between synthetic and observed seismograms is generally excellent. These results illustrate that the 3–D Earth model and the earthquake rupture model that we have used in this simulation is accurate enough to model seismic wave propagation on a global scale with periods of 5 sec and longer. Because the rupture duration of this event is more than 500 sec, the first arrival P waveform overlapped with the surface reflected wave of P-wave, which is called PP wave. Although this effect obscures the analysis of

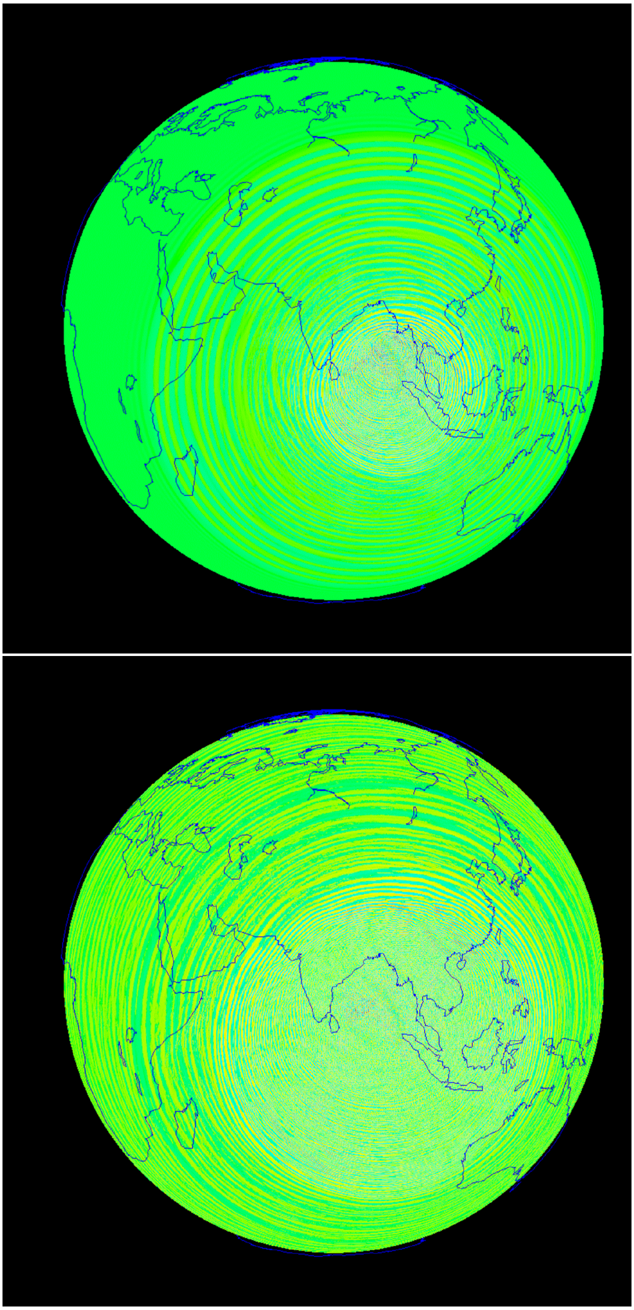

Fig. 3. Snapshots of the propagation of seismic waves excited by the December 26, 2004 Sumatra earthquake. Total displacement at the surface of the Earth is plotted at 10 min after the origin time of the event (top) and at 20 min after the origin time (bottom).

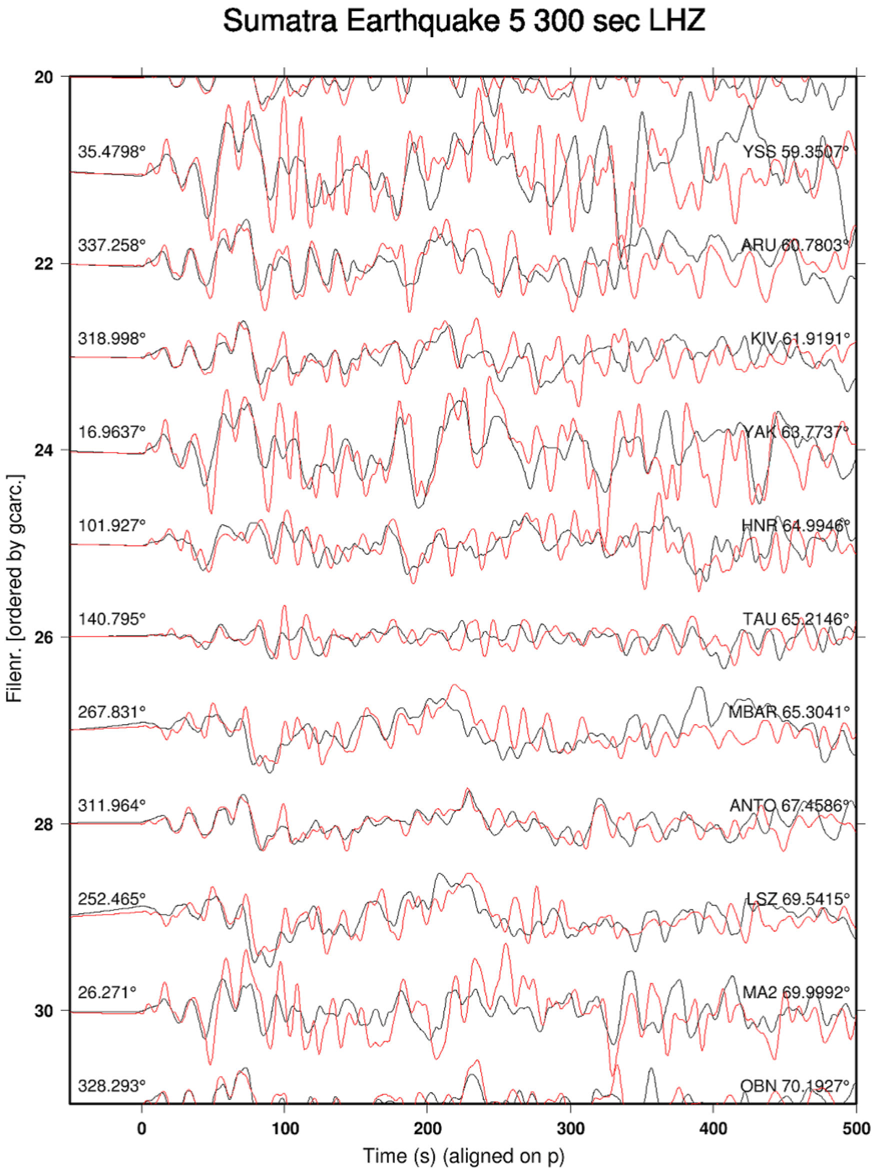

Fig. 4. Broadband data and synthetic displacement seismograms for the 2004 Sumatra earthquake, bandpass-filtered with a two-pass four-pole Butterworth filter between periods of 5 and 150 seconds. Vertical component data (black) and synthetic (red) displacement seismograms aligned on the arrival time of the P wave. For each set of seismograms the azimuth is plotted above the records to the left, and the station name and epicentral distance are plotted to the right.

earthquake source mechanism, it has been shown that the synthetic seismograms computed with Spectral-Element Method on the Earth Simulator can fully take these effects into account and are quite useful to study source mechanisms of this complicated earthquake.

4 Implications for the Earth's Internal Structure

The Earth's internal structure is another target that we can study by using our synthetic seismograms calculated for fully 3–D Earth model. We describe the examples of Tono et al (2005) [16]. They used records of $\sim$ 500 tiltmeters of the Hi-net, in addition to $\sim$ 60 broadband seismometers of the F-net, operated by the National Research Institute for Earth Science and Disaster Prevention (NIED). They analyzed pairs of sScS waves, which means that the S-wave traveled upward from the hypocenter reflected at the surface and reflected again at the core-mantle boundary, and its reverberation from the 410- or 660-km reflectors (sScSSdS where d=410 or 660 km) for the deep shock of the Russia-N.E. China border (PDE; 2002:06:28; 17:19:30.30; 43.75N; 130.67E; 566 km depth; 6.7 Mb). The two horizontal components are rotated to obtain the transverse component.

They have found that these records show clearly the near-vertical reflections from the 410- and 660-km seismic velocity discontinuities inside the Earth as post-cursors of sScS phase. By reading the travel time difference between sScS and sScSSdS, they concluded that this differential travel time anomaly can be attributed to the depth anomaly of the reflection point, because it is little affected by the uncertainties associated with the hypocentral determination, structural complexities near the source and receiver and long-wavelength mantle heterogeneity. The differential travel time anomaly is obtained by measuring the arrival time anomaly of sScS and that of sScSSdS separately and then by taking their difference. The arrival time anomaly of sScS (or sScSSdS) is measured by cross-correlating the observed sScS (or sScSSdS) with the corresponding synthetic waveform computed by SEM on the Earth Simulator. They plot the measured values of the two-way near-vertical travel time anomaly at the corresponding surface bounce points located beneath the Japan Sea. The results show that the 660-km boundary is depressed at a constant level of $\sim$ 15 km along the bottom of the horizontally extending aseismic slab under southwestern Japan. The transition from the normal to the depressed level occurs sharply, where the 660-km boundary intersects the bottom of the obliquely subducting slab. This observation should give important imprecations to geodynamic activities inside the Earth.

5 Conclusions

We have shown that the use of both the Earth Simulator and the SEM has allowed us to reach unprecedented resolution for the simulation of global seismic wave propagation resulting from large earthquakes. We have successfully attempted for the first time an independent validation of an existing 3–D Earth model. Such 3–D calculations on the Earth Simulator reach shorter periods than quasi-analytical 1-D spherically-symmetric solutions that are current practice in seismology. By using the SEM synthetics calculated for a realistic 3–D Earth model, it is possible to determine dif-ferences in the arrival times between theoretical seismograms and observations. As we have discussed in the present paper, these differences in arrival time can be interpreted as depth variations of the discontinuities. This kind of study would not have been possible without the combination of a precise seismic wave modeling technique, such as the SEM, on a powerful computer, such as the Earth Simulator, and a dense seismic observation network. If we extrapolate the numbers we used for our simulation, it is expected that we will get synthetic seismograms that are accurate up to 1 second for fully 3D Earth if we can use 64,896 CPUs of the Earth Simulator.

Acknowledgments. All the simulations were performed at the Earth Simulator Center of JAMSTEC by S.T. Broadband seismograms used in this study were recorded at Global Seismic Network stations and were obtained from the IRIS Data Management Center (www.iris.washington.edu) and F-net and Hi-net stations operated by the National Institute of Earth Science and Disaster Prevention.

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