# Simulations of Global Seismic Wave Propagation for 3-D Earth Model

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# ABSTRACT

We use a Spectral-Element Method implemented on the Earth Simulator in Japan to simulate broadband seismic waves generated by various earthquakes. The spectral-element method is based on a weak formulation of the equations of motion and has both the flexibility of a finite-element method and the accuracy of a pseudospectral method. The method has been developed on a large PC cluster and optimized on the Earth Simulator. We perform numerical simulation of seismic wave propagation for a three-dimensional Earth model, which incorporates 3D variations in compressional wave velocity, shear-wave velocity and density, attenuation, anisotropy, ellipticity, topography and bathymetry, and crustal thickness. The simulations are performed on 4056 processors, which require 507 out of 640 nodes of the Earth Simulator. We use a mesh with 206 million spectral-elements, for a total of 13.8 billion global integration grid points (i.e., almost 37 billion degrees of freedom). We show examples of simulations for several large earthquakes and discuss future applications in seismological studies.

#### 1. Introduction

Accurate modeling of seismic wave propagation in fully three-dimensional (3-D) Earth models is of considerable interest in seismology in order to determine both the 3-D seismic-wave velocity structure of the Earth and the rupture process during large earthquakes. However, significant deviations of Earth's internal structure from spherical symmetry, such as the 3-D seismic-wave velocity structure inside the solid mantle and laterally heterogeneous crust at the surface of the Earth, have made applications of analytical approaches to this problem a formidable task. The numerical modeling of seismic-wave propagation in 3-D structures has been significantly advanced in the last few years due to the introduction of the Spectral-Element Method (SEM), which is a high-degree version of the finite-element method that is very accurate for linear hyperbolic problems such as wave propagation. The 3-D SEM was first used in seismology for local and regional simulations [1-3], and more recently adapted to wave propagation at the scale of the full Earth [4-7].

Here we show that our implementation of the SEM on the Earth Simulator in Japan allows us to calculate theoretical seismic waves which are accurate up to 3.5 seconds and longer for fully 3-D Earth models. We include the full complexity of the 3-D Earth in our simulations, i.e., a 3-D seismic wave velocity [8] and density structure, a 3-D crustal model [9], ellipticity as well as topography and bathymetry. Synthetic waveforms at such high resolution (periods of 3.5 seconds and longer) allow us to perform direct comparisons of arrival times of various body-wave phases between observed and synthetic seismograms, which has never been accomplished before. Usual seismological algorithms, such as normal-mode summation techniques that calculate quasi-analytical synthetic seismograms for one-dimensional (1-D) spherically symmetric Earth models [10], are typically accurate down to 8 seconds [11]. In other words, the SEM on the Earth Simulator allows us to simulate global seismic wave propagation in fully 3-D Earth models at periods shorter than current seismological practice for simpler 1-D spherically symmetric models.

The results of our simulation show that the synthetic seismograms calculated for fully 3-D Earth models by using the Earth Simulator and the SEM agree well with the observed seismograms, which illustrates that the current 3-D seismic

velocity model captures the general long-wavelength image of Earth's interior with sufficient resolution.

### 2. Spectral-element method

We use the spectral-element method (SEM) developed by Komatitsch and Tromp (2002a, 2002b) [5,6] to simulate global seismic wave propagation throughout a 3-D Earth model, which includes a 3-D seismic velocity and density structure, a 3-D crustal model, ellipticity as well as topography and bathymetry. The SEM first divides the Earth into six chunks. Each of the six chunks is divided into slices. Each slice is allocated to one CPU of the Earth Simulator. Communication between each CPU is done by MPI. Before the system can be marched forward in time, the contributions from all the elements that share a common global grid point need to be summed. Since the global mass matrix is diagonal, time discretization of the second-order ordinary differential equation is achieved based upon a classical explicit second-order finite-difference scheme.

The maximum number of nodes we could use for this simulation is 4056 processors, i.e., 507 nodes out of 640 of the Earth Simulator. Each slice is allocated to one processor of the Earth Simulator and subdivided with a mesh of 48× 48 spectral-elements at the surface of each slice. Within each surface element we use  $5 \times 5 = 25$  Gauss-Lobatto-Legendre (GLL) grid points to interpolate the wave field [12, 13], which translates into an average grid spacing of 2.0 km (i.e., 0.018 degrees) at the surface. The total number of spectral elements in this mesh is 206 million, which corresponds to a total of 13.8 billion global grid points, since each spectral element contains  $5 \times 5 \times 5 = 125$  grid points, but with points on its faces shared by neighboring elements. This in turn corresponds to 36.6 billion degrees of freedom (the total number of degrees of freedom is slightly less than 3 times the number of grid points because we solve for the three components of displacement everywhere in the mesh, except in the liquid outer core of the Earth where we solve for a scalar potential). Using this mesh, we can calculate synthetic seismograms that are accurate down to seismic periods of 3.5 seconds. This simulation uses a total of approximately 7 terabytes of memory. Total performance of the code, measured using the MPI Program Runtime Performance Information was 10 teraflops, which is about one third of the

expected



Fig. 1 The SEM uses a mesh of hexahedral finite elements on which the wave field is interpolated by high-degree Lagrange polynomials on Gauss-Lobatto-Legendre (GLL) integration points. This figure shows a global view of the mesh at the surface, illustrating that each of the six sides of the so-called `cubed sphere' mesh is divided into 26 X 26 slices, shown here with different colors, for a total of 4056 slices (i.e., one slice per processor).



Honshu November 12 2003 BHZ

Fig. 2 Broadband data and synthetic displacement seismograms for the 2003 South of Honshu earthquake bandpass-filtered with a two-pass four-pole Butterworth filter between periods of 3.5 and 150 seconds. Vertical component data (black) and synthetic (red) displacement seismograms aligned on the arrival time of the P wave are shown. For each set of seismograms the azimuth is printed above the records to the left, and the station name and

epicentral distance are printed to the right.

peak performance for this number of nodes (507 nodes × 64gigaflops = 32 teraflops). Figure 1 shows a global view of the spectral-element mesh at the surface of the Earth. In Figure 2, we compare the vertical component of displacement from synthetic seismograms calculated using 507 nodes of the Earth Simulator and observed records for several broadband seismic stations of the F-net array operated by the National Institute of Earth Science and Disaster Prevention in Japan. The earthquake we simulated is a deep earthquake of magnitude 6.3 that occurred in South of Japan on November 12, 2003, at a depth of 382 km. It is suprising that the global 3-D seismic velocity model used in this simulation still produces fairly good agreement with the observations even at periods of 3.5 seconds, because it is supposed that the crustal and mantle structure beneath Japanese Islands are highly heterogeneous and are not captured by the long-wavelength global 3D Earth model. However, Figure 2 also shows that the theoretical seismograms calculated with 507 nodes of the Earth Simulator do not reproduce some of the fine features in the observation and suggests the limitation of this global 3-D seismic velocity model. For those stations located to the north-east of the epicenter (the azimuth is about 20 degrees), the observed waves show large high-frequency oscillations because the waves travel along the subducting pacific plate, but this feature is not modeled in the theoretical seismograms. This shows that we need to improve our 3-D seismic wave velocity model to calculate theoretical seismic waves that are accurate at 3.5 seconds and longer.

#### 3. Simulation of the 2002 Denali fault earthquake

Because we have found that we do not have a 3-D Earth model which has sufficient resolution to simulate seismic wave propagation accurately in regional scale, we decide to use 243 nodes (1944 CPUs) of the Earth Simulator for the simulation using the SEM. Using 243 nodes (1944 CPUs), we can subdivide the six chunks into 1944 slices (1944=6×18×18). Each slice is then subdivided into 48 elements in one direction. Because each element has 5 Gauss-Lobatto Legendre integration points, the average grid spacing at the surface of the Earth is about 2.9 km. The number of grid points in total amounts to about 5.5 billion. Using this mesh, it is expected that we can calculate synthetic seismograms accurate up to 5 sec all over the globe. For the 243 nodes case, the total performance we achieved was about 5 teraflops, which also is about one third of the peak performance. The fact that when we double the number of nodes from 243 to 507 the total performance also doubles from 5 teraflops to 10 teraflops shows that our SEM code exhibits an excellent scaling relation with respect to performance. We calculate synthetic seismograms for a 3-D Earth model using the SEM code and 243 nodes of the ES for the November 3, 2002 Denali, Alaska earthquake (Mw 7.9, depth 15.0 km).

The November 3, 2002 Denali, Alaska earthquake is the largest strike-slip event in North America in almost 150 years. The earthquake started its rupture with a small thrust event and propagated in a southeastern direction along 220 km of the Deanli fault [14, 15]. To simulate synthetic seismograms for this earthquake, we represent the earthquake source by 475 point sources distributed both in space and time [16]. In Figure 3, we show snapshots of seismic wave propagation along the surface of the Earth. Because the rupture along the fault propagated in a south-easterly direction, the seismic waves radiated in this direction are strongly amplified. This is referred as the directivity caused by the earthquake source mechanisms. Figure 3 illustrate that the amplitude of the seismic waves becomes



Fig. 3 Snapshots of the propagation of seismic waves excited by the November 3, 2002 Denali fault earthquake. Total displacement at the surface of the Earth is plotted at 4 min 20 sec after the origin time of the event (top) and at 17 min 20 sec after the origin time (bottom).

large along the west coast of the United States and shows that this directivity is modeled well. Because there are more than 200 seismographic observatories, which are equipped with broadband seismometers all over the globe, we can directly compare the synthetic seismograms calculated with the Earth Simulator and the SEM with the observed seismograms. Figure 4 shows the results of this comparison for vertical ground motion and demonstrates that the agreement between synthetic and observed seismograms is generally excellent. These results illustrate that the 3-D Earth model that we have used in this simulation is accurate enough to model seismic wave propagation on a global scale with periods of 5 sec and longer. The results of this simulation is summarized in Tsuboi et al (2003) [17] and Komatitsch et al (2003) [18], which was awarded 2003 Gordon Bell prize for peak performance in SC2003.



Fig. 4 Broadband data and synthetic displacement seismograms for the 2002 Alaska earthquake, bandpass-filtered with a two-pass four-pole Butterworth filter between periods of 5 and 150 seconds. Vertical component data (black) and synthetic (red) displacement seismograms aligned on the arrival time of the P wave. For each set of seismograms the azimuth is plotted above the records to the left, and the station name and epicentral distance are plotted to the right.

### 4. Simulation of the 2003 Tokachi-Oki earthquake

We also simulate seismic wave propagation excited by a recent large earthquake which occurred near the Japanese Islands on September 25, 2003. The fault plane solution of this Mw=8.3 (Harvard CMT) September 25, 2003 Tokachi-Oki earthquake is a shallow dipping thrust fault, which shows that this event is a typical inter-plate large earthquake along the subducting pacific plate (e.g., Yamanaka and Kikuchi, 2003 [19]). It is generally believed that the 2003 Tokachi-Oki earthquake was a recurrence of the M=8.2 (JMA) March 4, 1952 Tokachi-Oki earthquake, because of its proximity in rupture area and similarity of earthquake source mechanism. We first reconstruct the slip history, slip vector, rupture initiation time, and width of an analytic slip-rate function on each sub-fault (Ji et al., in preparation, 2004). The finite rupture along the fault plane is modeled by 870 sub-events of size 6km×5 km, which represents the distribution of the moment-density tensor along the subducting Pacific plate. The fault plane is subdivided by two fault segment. Based on the results of inversion, we take 745 sub-events with scalar moment larger than 1.5e+17 Nm and plot



the slip distribution along the entire fault (Fig. 5). The slip distribution and history indicate that the largest slip of more than 7 m occurred in the east of Cape Erimo centered at 143.5E and 42.2N about 20 sec after the rupture started. The largest slip occurs in the region, where the previous large event in 1952 supposedly occurred.



Figure 5. Slip distribution along the fault plane of September 25, 2003, Tokachi-Oki earthquake. The black star denotes the point of rupture initiation. Black triangles in Hokkaido Island indicate strong motion seismographic stations operated by the National Institute for Earth Science and Disaster Prevention, of which seismograms are shown in Figure 6.

The simulation of seismic wave propagation for this event is performed in the same manner for the 2002 Denali fault earthquake. We use 243 nodes of the Earth Simulator with the above 745 point sources as earthquake source parameters. Because there are a huge number of seismic stations in the Japanese Islands, we can compare the synthetics seismograms computed for a realistic global 3-D Earth model with these observed seismograms. Here we try to compare some of strong motion seismographic stations operated by the National Institute for Earth Science and Disaster Prevention. which are situated very close to the earthquake source. The locations of these seismic stations are shown in Fig. 5. We show the comparison in Fig. 6. Because we use a global three-dimensional Earth model in our simulation, the agreement is not as good as we see for teleseismic stations, but it should be noted that the arrival time and the amplitude of first arrival phases match reasonably well for those stations which are located just above the fault surface, such as HKD112 and HKD113. This result illustrates that we need a precise 3-D crust model for regions like Japan, where the crustal structure is very heterogeneous due to the subducting tectonic plate. There are many seismic surveys around the Japanese Islands to determine shallow crustal structure including sedimentary layers. We are now trying to compile these surveys to construct a complete 3-D crustal model for the Japanese Islands. We believe that including this 3-D crustal structure with 3-D tomographic velocity model and 3-D plate geometry will significantly improve our seismic wave simulations around the Japanese Islands.

Figure 7 shows the snapshots of propagation of seismic waves in the Japanese Islands excited by the Tokachi-Oki earthquake. Because the earthquake rupture propagates in northwestern direction from the hypocenter, this figure also shows the large amplitude in this direction. There was significant damage along the coast of the western part of Hokkaido Island for this earthquake. The source of directivity reproduced in Figure 7 may explain some of the damage caused by the large amplitude surface waves generated by this earthquake and shows the importance of including finite source in the seismic wave modeling..



Figure 6. Vertical component strong motion data and synthetic velocity seismograms bandpass-filtered with a two-pass four-pole Butterworth filter between periods of 5 and 150 seconds. Both data and synthetics are bandpass-filtered and then converted to velocity. Data are shown in black and synthetics are in red and aligned on the arrival time of the P wave. For each set of seismograms the azimuth and the peak velocity are plotted above the records to the right, and the station name is plotted to the left. The positions of these stations are shown in Fig. 5.

#### 5. Implications for the Earth's internal structure

The Earth's internal structure is another target that we can study by using our synthetic seismograms calculated for fully 3-D Earth model. We describe the examples of Tono et al (2003) [20]. They used records of ~500 tiltmeters of the Hi-net, in addition to ~60 broadband seismometers of the F-net, operated by the National Research Institute for Earth Science and Disaster Prevention (NIED). They analyzed pairs of sScS waves, which means that the S-wave traveled upward from the hypocenter reflected at the surface and reflected again at the core-mantle boundary, and its reverberation from the 410- or 660-km reflectors (sScSSdS where d=410 or 660 km) for the deep shock of the Russia-N.E. China border (PDE; 2002:06:28; 17:19:30.30; 43.75N; 130.67E; 566 km depth; 6.7 Mb).

The two horizontal components are rotated to obtain the transverse component. They have found that these records show clearly the near-vertical reflections from the 410- and 660-km seismic velocity discontinuities inside the Earth as post-cursors of



sScS phase. By reading the travel time difference between sScS



Fig. 7 Snapshots of the propagation of seismic waves excited by the September 25, 2003 Tokachi-Oki earthquake. Total displacement at the surface of the Earth is plotted at 1 min 40 sec, 3 min 20 sec, 5 min. after the origin time, from top to bottom. and sScSSdS, they concluded that this differential travel time anomaly can be attributed to the depth anomaly of the reflection point, because it is little affected by the uncertainties associated with the hypocentral determination, structural complexities near the source and receiver and long-wavelength mantle heterogeneity. The differential travel time anomaly is obtained by measuring the arrival time anomaly of sScS and that of sScSSdS separately and then by taking their difference. The arrival time anomaly of sScS (or sScSSdS) is measured by cross-correlating the observed sScS (or sScSSdS) with the corresponding synthetic waveform computed by SEM on the Earth Simulator. They plot the measured values of the two-way near-vertical travel time anomaly at the corresponding surface bounce points located beneath the Japan Sea. The results show that the 660-km boundary is depressed at a constant level of ~15 km along the bottom of the horizontally extending aseismic slab under southwestern Japan. The transition from the normal to the depressed level occurs sharply, where the 660-km boundary intersects the bottom of the obliquely subducting slab. This observation should give important imprecations to geodynamic activities inside the Earth.

# 6. Conclusions

We have shown that the use of both the Earth Simulator and the SEM has allowed us to reach unprecedented resolution for the simulation of global seismic wave propagation resulting from large earthquakes. We have successfully attempted for the first time an independent validation of an existing 3-D Earth model. Such 3-D calculations on the Earth Simulator reach shorter periods than quasi-analytical 1-D spherically-symmetric solutions that are current practice in seismology. By using the SEM synthetics calculated for a realistic 3-D Earth model, it is possible to determine differences in the arrival times between theoretical seismograms and observations. As we have discussed in the present paper, these differences in arrival time can be interpreted as depth variations of the discontinuities. This kind of study would not have been possible without the combination of a precise seismic wave modeling technique, such as the SEM, on a powerful computer, such as the Earth Simulator, and a dense seismic observation network.

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