High-order multi-step one-way modeling

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Abstract

This work concerns numerical aspects of a one-way formulation of the acoustic wave equation. We focus our study on a system of coupled one-way equations which generalizes the multi-step one-way modelling proposed by Kiyashchenko, Plessix and Kashtan. We show how the performance of the numerical method can be improved by developping a fast numerical algorithm whose efficiency is illustrated by some numerical experiments.

Introduction

The Reverse Time Migration (RTM) is an efficient method for depth imaging. RTM is based upon successive solutions of the wave equation and it obviously depends on the accuracy of the numerical solution of the wave equation but also on the computational burden which must be the lowest as possible to be applied to 3D problems in heterogeneous media. The migration process uses sismograms which involve two quantities: the arrival times and the amplitudes of the reflected waves which respectively represent the kinematics and the dynamics of the propagation medium. Solving one-way equations provide a fast solution for the acoustic wave equation which allows one to reproduce the kinematics but the amplitudes of the wave fields are generally erroneous because the one-way model neglects coupling terms modeling the transmission effects between the different materials constituting the propagation medium. Zhang et al. [6] have proposed an approximate formulation including an additional term to correct the amplitude of the solution. More recently, Kiyashchenko et al. [4] have proposed a multi-step one-way modelling which is equivalent to the wave equation. Herein we focus our attention on a first-order formulation which has been derived by M.V. De Hoop [2] in the framework of micro-local analysis.

The complete first-order formulation of the acoustic wave equation

The wave equation can be written as a first-order system of coupled equations after the time variable has been supressed by using a Laplace transform. The principal part of the system can be diagonalized and the reduced system involves then pseudo-differential operators. In practice, the numerical solution is obtained by solving an approximation of the reduced system in which the pseudo-differential operators have been replaced by their principal part. This is the system we consider and describe below.

Let x, y, z be the cartesian coordinates. Let us consider a domain Ω in $z > 0$ whose surface is given by the set $z = 0$. Then the first-order wave equation system reads as:

$$\left(\mathbf{D}_z + i\omega\Lambda\right)\mathbf{V} = R\mathbf{V} + P\mathbf{F} \text{ in } \Omega \tag{1}$$

where Λ is diagonal with $\Lambda = diag(\Gamma, -\Gamma)$ and Γ is a pseudo-differential operator whose symbol γ is given by:

$$\gamma = \left(\frac{1}{c^2\left(\vec{x}\right)} - \frac{\left|\vec{k}'\right|^2}{\omega^2} \right)^{1/2} . \tag{2}$$

Operator R represents the coupling terms describing the reflexion and transmission phenomena and has the form:

$$R = \begin{pmatrix} T & R_{du} \\ R_{ud} & T \end{pmatrix} \tag{3}$$

The vector F is defined from the source S, for instance S is a Ricker function acting at $z = 0$. The unknown $\mathbf{V}$ has two components respectively denoted by V_d and V_u which respectively propagate in the direction $z > 0$ and in the opposite sense. Hence V_d is the downward part of the wave field while V_u is its upward part. The operator P is supposed to be invertible and allows one to construct the solution $\mathbf{U}$ to the wave equation from the relation $\mathbf{V} = P\mathbf{U}$. Moreover, P can be chosen such that $T = -R_{du} = -R_{ud}$. The operator T represents the transmission effects while R_{du} and R_{ud} correspond to the reflections. The transmission operator is defined as the principal part of $-\frac{1}{2}\Gamma^{-1}\frac{\partial T}{\partial z}$ which means that the symbol of T is given by:

$$\sigma\left(T\right) = \frac{\omega^2}{2c^3\gamma^2}\frac{\partial c}{\partial z} \tag{4}$$

The above formula shows that when the medium is homogeneous, T is the null operator since the velocity c does

not vary. Thus in that case, the two components of $\mathbf{V}$ are uncoupled and satisfy a one-way system, involving the square-root of the Helmholtz operator in Γ. In the general case, the two components are coupled through R_{du} and R_{ud} and since the symbol of T is real-valued, T affects the amplitude of each component of $\mathbf{V}$. Hence if T is neglected, the dynamics is erroneous.

Numerical scheme

System 1 can be solved by using different approaches. Here we choose to expand the solution $\mathbf{V}$ as a Neumann series and to compare our method to the one formerly proposed by [4]. Assume that the inverse G of $\mathbf{D}_z + i\omega\Lambda$ is known. Then V is given by:

$$(I - GR)\, V = GF \tag{5}$$

Next the formal inverse of $I - GR$ is represented by a Neumann series and we have:

$$\mathbf{V} = \textstyle\sum_{j\geq 0} \mathbf{V}_j \mathbf{V}_0 = GF \text{ and } \mathbf{V}_j = GR\mathbf{V}_{j-1} \tag{6}$$

The first iterate $\mathbf{V}_0$ is obtained by solving two uncoupled one-way equations and models the propagation of the source F. The iterate $\mathbf{V}_1$ corrects $\mathbf{V}_0$ by accounting for the reflection and transmission terms. It is solution to :

$$(\mathbf{D}_z + i\omega\Lambda)\, \mathbf{V}_1 = R\mathbf{V}_0 \tag{7}$$

and the iterate $\mathbf{V}_j, j \geq 2$ is obtained by solving the same problem as above with $\mathbf{V}_j$ in place of $\mathbf{V}_1$ and $\mathbf{V}_{j-1}$ in place of $\mathbf{V}_0$.

The numerical approximation of $\mathbf{V}$ is defined by computing a finite number of iterates $\mathbf{V}_j$ and according to [5], the performances of the numerical algorithm can be improved by using an assembling process allowing one to compute two iterates in the same time. In [5], both G and R are represented by Fourier integrals and to limit the computational burden, their respective symbol are approximated by a class of functions where k' and (x, y) are separate. Then the number of required Fourier transforms decreases considerably.

In practice, the number of iterates is fixed at the beginning and it is not necessary to compute a lot of terms to obtain a high degree of accuracy.

Here we assume the upward part of $\mathbf{V}_0$ is null which means the region $z < 0$ behaves like the free space. Hence the propagation of the source involves the first one-way equation only. To compute the next terms, it is necessary to solve the two one-way equations after the right-hand side has been computed. Any entry of R is equal to $\pm T$ and T involves $\partial_z \Gamma$. By definition of Γ, we have:

$$\partial_z \Gamma = \frac{\omega^2 \partial_z c}{2c^3} \Gamma^{-1} \tag{8}$$

which implies that T acts like the principal part of

$$\frac{\omega^2 \partial_z c}{2c^3} \Gamma^{-2} \tag{9}$$

We can then observe that T involves the inverse of the Helmholtz equation.

We now compare our approach to the one proposed by Kiyashchenko *et al.* [4] for solving the scalar wave equation. To get the same type of numerical scheme, it is necessary to include the diagonal entries of R into the system of one-way equations and thus, the left-hand side of the system involves the extra terms of R only. We then get the same type of numerical scheme than in [4] by changing Γ into the identity and T by its Padé approximation at high-frequency. To replace Γ by the identity amounts to consider the simplest high-frequency approximation. Hence we can say that the numerical scheme in [4] corresponds to a high-frequency approximation of our scheme. Moreover, the solution computed in [4] corresponds to the sum of $\mathbf{V}_0$ and $\mathbf{V}_1$ only while we can consider high-order terms which do not require a high computational cost by using the assembling process suggested in [5]. This is why we claim that we generalize the approach in [4].

Illustrations

In that section, we intend to illustrate the performance of our numerical scheme by considering a synthetic 2D velocity model, the so-called GXT model. We represent the arrival times and we compare our results with the ones obtained by a finite element method. We use the SPECFEM2D software.

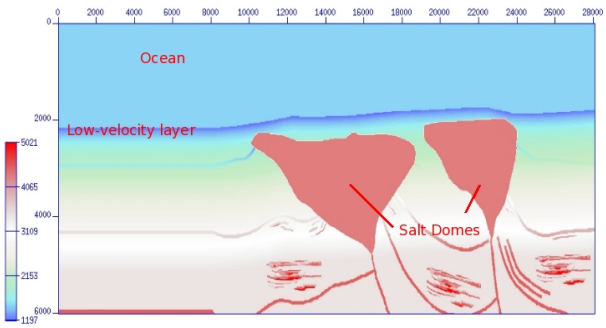

Figure 1: Velocity model GXT

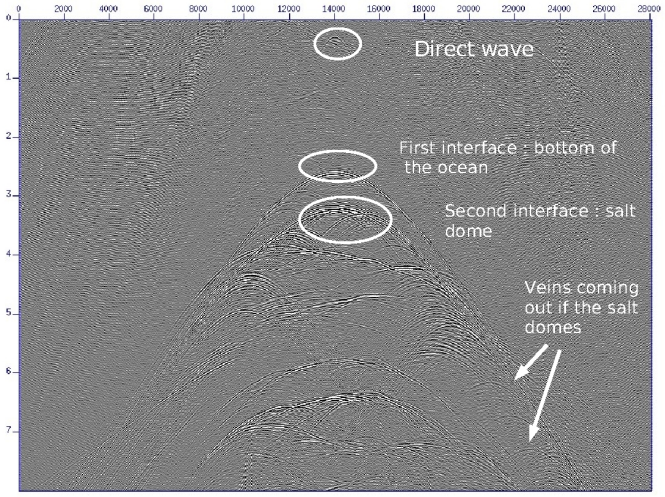

Figure 2: Sismogram obtained with the multi-step one-way system

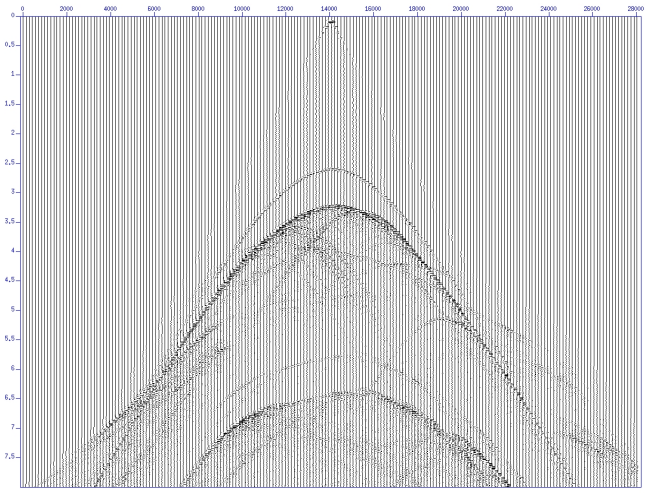

Figure 3: Sismogram obtained with the specfem2D software

The model is composed of a top layer with relatively low-velocity (water), a low-velocity layer (bottom of the ocean), and two salt domes. There is a velocity gradient, increasing as it goes deeper. The model is 28 x 6 kilometers, the source time function is a Ricker centered at 30 Hz, located at the middle of the top of the model, and the receivers, located along the top of the model, recorded 8 seconds. We focus our attention on the kinematics because we intend to apply migrations techniques. We can then observe that the sismograms are in good agreement and that the main interfaces such as the bottom of the ocean, and the first salt dome can be identified.

The next step of investigation concerns the application of migration techniques based on our solver for the wave equation. At present time, we are considering different methods which will be presented and compared to complete this work.

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